Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number	Candidate Number

Tuesday 19 January 2021

Morning (Time: 1 hour 30 minutes)	Paper Reference **WME01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M1

You must have: Mathematical Formulae and Statistical Tables (Blue), calculator	Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}, and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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1. A small stone is projected vertically upwards with speed $20\,\mathrm{m\,s^{-1}}$ from a point O which is $5\,\mathrm{m}$ above horizontal ground. The stone is modelled as a particle moving freely under gravity.

Find

(a) the speed of the stone at the instant when it is $2\,\mathrm{m}$ above the ground, **(2)**

(b) the total time between the instant when the stone is projected from O and the instant when it first strikes the ground. **(4)**

a) The stone is moving freely under gravity therefore the acceleration is constant and SUVAT can be used.

s : −3 (change in height)
u : 20 (initial velocity)
v : v (velocity at height of 2m)
a : −9.8 (acceleration due to gravity)
t :

SUVAT formula without t :

$$\boxed{v^2 = u^2 + 2as}$$

$\therefore v^2 = 20^2 + 2(-9.8)(-3)$

$\therefore v^2 = 458.8 \quad \therefore v = \sqrt{458.8} \approx 21.4\,\mathrm{ms^{-1}}$ (3sf)

b) Set up a SUVAT for the entire journey.

s : −5 (total displacement from start to end)
u : 20 (initial velocity)
v :
a : −9.8 (acceleration due to gravity)
t : t (total time)

SUVAT formula without v :

$$\boxed{s = ut + \tfrac{1}{2}at^2}$$

$\therefore -5 = 20t + \frac{1}{2}(-9.8)t^2 \quad \therefore -4.9t^2 + 20t + 5 = 0$

$\therefore t = 4.31795\ldots$ OR $t = -0.23631\ldots$

Time is scalar so can't be negative $\therefore t = 4.32$ seconds // (3sf)

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Question 1 continued

Q1

(Total 6 marks)

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2. Two particles, P and Q, have masses $2m$ and m respectively. The particles are moving towards each other in opposite directions along the same straight line on a smooth horizontal plane. The particles collide directly.

Immediately before the collision, the speed of P is $3u$ and the speed of Q is $2u$.

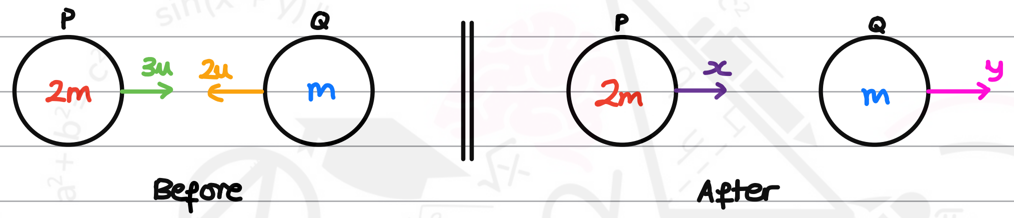
The magnitude of the impulse exerted on Q by P in the collision is $5mu$.

Find

(a) the speed of P immediately after the collision, (labelled x) **(3)**

(b) the speed of Q immediately after the collision. (labelled y) **(3)**

a)b) Draw a diagram labelling the masses and speeds.



Using conservation of linear momentum : $\boxed{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2}$

$\therefore\ 2m(3u) + m(-2u) = 2m(x) + m(y)$

$\therefore\ 6mu - 2mu = 2mx + my$

$\therefore\ 4u = 2x + y$

Using the formula for impulse : $\boxed{I = mv - mu}$

$\therefore\ 5mu = m(y) - m(-2u)$

$\therefore\ 5mu = my + 2mu$

$\therefore\ y = \dfrac{5mu - 2mu}{m} = 3u$ //

Substitute this back into the equation above.

$\therefore\ 4u = 2x + y$

$\therefore\ 4u = 2x + 3u \quad \therefore\ 2x = u \quad \therefore\ x = \frac{1}{2}u$ //

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Question 2 continued

Q2

(Total 6 marks)

3.

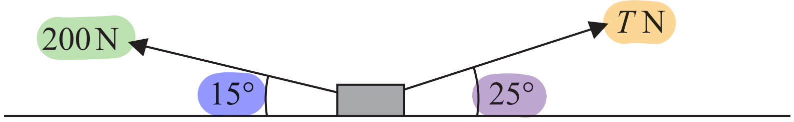


Figure 1

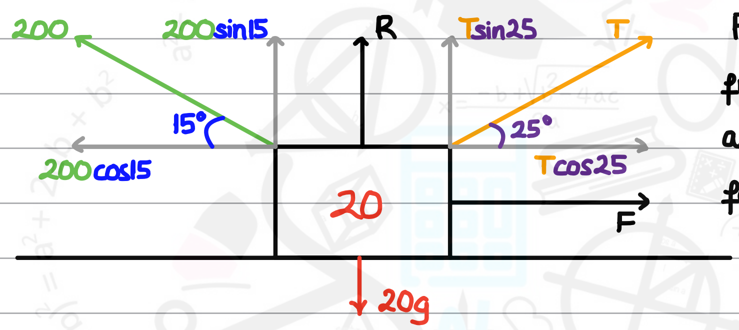
A parcel of mass 20 kg is at rest on a rough horizontal floor. The coefficient of friction between the parcel and the floor is 0.3

Two forces, both acting in the same vertical plane, of magnitudes 200 N and T N are applied to the parcel. The line of action of the 200 N force makes an angle of 15° with the horizontal and the line of action of the T N force makes an angle of 25° with the horizontal, as shown in Figure 1. The parcel is modelled as a particle P.

Find the smallest value of T for which P remains in equilibrium.

(9)

Draw a diagram labelling all relevant forces.



For T to be at a minimum, friction must act to the right as the sum of the horizontal forces is zero.

The parcel is at rest meaning the sum of the horizontal and vertical forces is zero.

Vertically : $\boxed{\Sigma F = 0}$ $\therefore R + 200\sin 15 + T\sin 25 - 20g = 0$

$\therefore R = 20g - 200\sin 15 - T\sin 25$

$R = 20(9.8) - 200(0.258...) - T(0.422...)$

$R = 144.236... - 0.423T$

Horizontally : $\boxed{\Sigma F = 0}$ $\therefore T\cos 25 + F - 200\cos 15 = 0$

Friction formula : $\boxed{F = \mu R}$ $\therefore T\cos 25 + 0.3R - 200\cos 15 = 0$

Substituting R in : $T\cos 25 + 0.3(144.236... - 0.423T) - 200\cos 15 = 0$

$\therefore 0.906...T + 43.270... - 0.126...T - 193.185... = 0$

$\therefore 0.779...T = 149.914$

$\therefore T = 192.444 \approx 192\text{N}$ (3sf)

P 6 1 9 0 2 A 0 6 2 4

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Question 3 continued

Q3

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4.

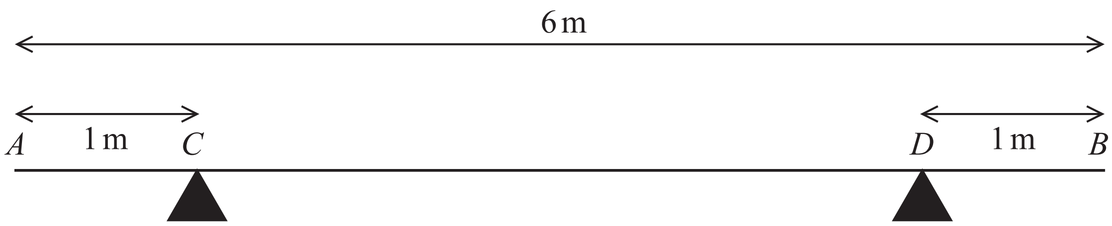


Figure 2

A metal girder AB has weight W newtons and length 6 m. The girder rests in a horizontal position on two supports C and D where $AC = DB = 1$ m, as shown in Figure 2.

① When a force of magnitude 900 N is applied vertically upwards to the girder at A, the girder is about to tilt about D.

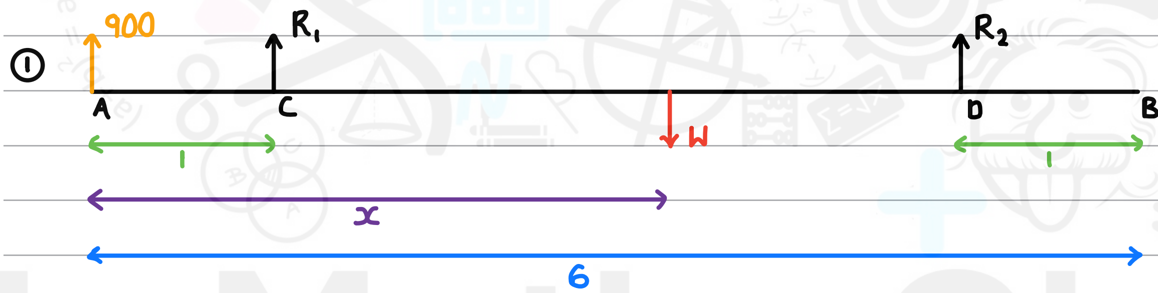
② When a force of magnitude 1500 N is applied vertically upwards to the girder at B, the girder is about to tilt about C.

The girder is modelled as a non-uniform rod whose centre of mass is a distance x metres from A.

Find the value of x.

(6)

Draw a diagram labelling the forces and lengths for each senario.



If the girder is about to tilt about D, the reaction force at C is zero ($R_1 = 0$).

Since it's stated in the question that the beam is in equilibrium, the sum of the clockwise moments is equal to the sum of the anticlockwise moments, therefore:

$\boxed{\Sigma \text{moments clockwise} = \Sigma \text{moments anticlockwise}}$

where $\boxed{\text{moment} = \text{force} \times \text{perpendicular distance}}$

P 6 1 9 0 2 A 0 8 2 4

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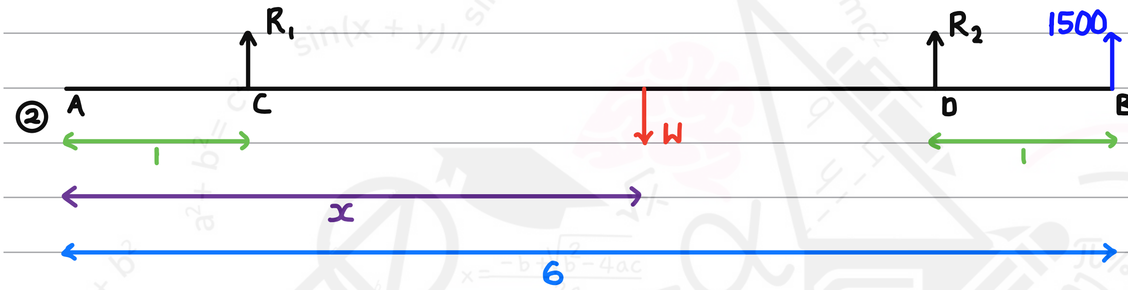
Question 4 continued

The clockwise forces are ones that go upwards from the left or downwards from the right of where moments are taken and anticlockwise forces are ones that go upwards from the right or downwards from the left of where moments are taken.

Taking moments about D :

$\underbrace{900(6-1)}_{\text{clockwise moments}} = \underbrace{W(6-1-x)}_{\text{anticlockwise moments}}$ $\quad \therefore 4500 = 5W - Wx$

$\therefore Wx = 5W - 4500$



If the girder is about to tilt about C, the reaction force at D is zero ($R_2 = 0$).

Taking moments about C :

$\underbrace{W(x-1)}_{\text{clockwise moments}} = \underbrace{1500(6-1)}_{\text{anticlockwise moments}}$ $\quad \therefore Wx - W = 7500$

$\therefore Wx = 7500 + W$

Combining both expressions of Wx :

$5W - 4500 = 7500 + W$

$\therefore 4W = 12000 \quad \therefore W = 3000\text{N}$

Substitute the value of W into either moment equation :

① $3000x = 5(3000) - 4500$

$x = \frac{10500}{3000} = 3.5\text{m}$ //

② $3000x = 7500 + 3000$

$x = \frac{10500}{3000} = 3.5\text{m}$ //

Q4

(Total 6 marks)

Turn over

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5. A particle is acted upon by two forces **F** and **G**. The force **F** has magnitude 8 N and acts in a direction with a bearing of 240°. The force **G** has magnitude 10 N and acts due South.

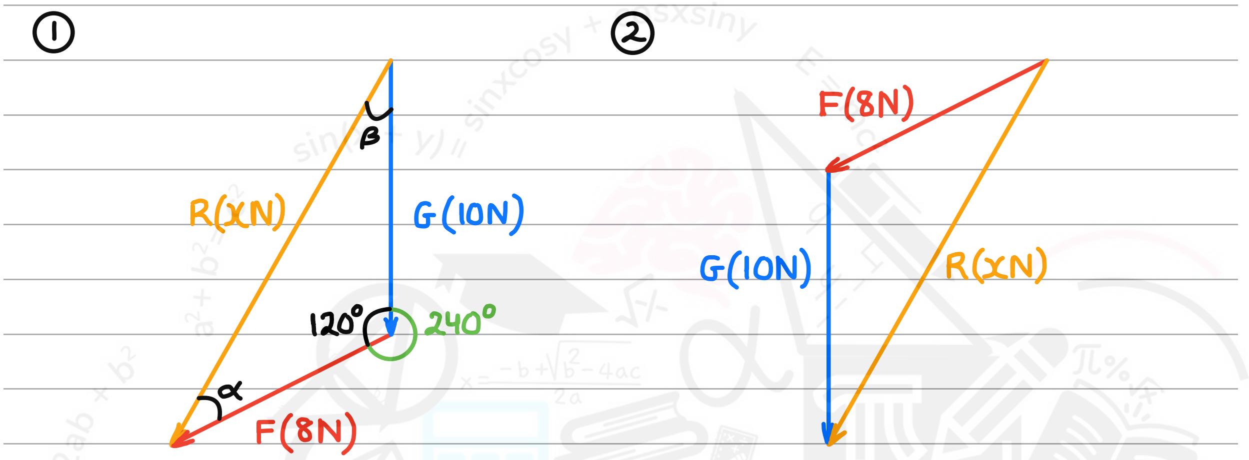
Given that **R** = **F** + **G**, find

(i) the magnitude of **R**,

(ii) the direction of **R**, giving your answer as a bearing to the nearest degree.

(7)

(i) Draw a diagram of the forces as a triangle where R is the resultant of F and G with a magnitude of x N.



The order of the forces F and G is irrelevant as they both have the same resultant with the same magnitude and direction so for simplicity we will use the first one as its easier to find one of the internal angles.

Using Cosine Rule : $\boxed{a^2 = b^2 + c^2 - 2bc(\cos A)}$

$x^2 = 8^2 + 10^2 - 2(8)(10)(\cos 120)$

$x^2 = 164 - 160\cos 120$

$x^2 = 164 - 160(-0.5)$

$x^2 = 244 \quad \therefore \quad x = \sqrt{244} \approx 15.6\text{ N}$ // (3sf)

(ii) Bearings are taken clockwise from north therefore the bearing of R is $\beta + 180°$.

Using Sine Rule : $\boxed{\dfrac{a}{\sin A} = \dfrac{b}{\sin B}}$ $\qquad \dfrac{x}{\sin 120} = \dfrac{8}{\sin \beta}$

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Question 5 continued

$\therefore \sin\beta = \dfrac{8\sin 120}{x} = \dfrac{8\left(\frac{\sqrt{3}}{2}\right)}{\sqrt{244}} = \dfrac{2\sqrt{183}}{61}$

$\therefore \beta = \sin^{-1}\left(\dfrac{2\sqrt{183}}{61}\right) = 26.329... \approx 26°$

$\therefore$ Bearing of R $= 26 + 180 = 206°$ // (nearest degree)

(Total 7 marks)

Q5

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6. Two girls, Agatha and Brionie, are roller skating inside a large empty building. The girls are modelled as particles.

At time $t = 0$, Agatha is at the point with position vector $(11\mathbf{i} + 11\mathbf{j})$ m and Brionie is at the point with position vector $(7\mathbf{i} + 16\mathbf{j})$ m. The position vectors are given relative to the door, O, and $\mathbf{i}$ and $\mathbf{j}$ are horizontal perpendicular unit vectors.

Agatha skates with constant velocity $(3\mathbf{i} - \mathbf{j})\,\text{m s}^{-1}$

Brionie skates with constant velocity $(4\mathbf{i} - 2\mathbf{j})\,\text{m s}^{-1}$

(a) Find the position vector of Agatha at time t seconds.

(2)

At time $t = 6$ seconds, Agatha passes through the point P.

(b) Show that Brionie also passes through P and find the value of t when this occurs.

(4)

At time t seconds, Agatha is at the point A and Brionie is at the point B.

(c) Show that $\overrightarrow{AB} = [(t-4)\mathbf{i} + (5-t)\mathbf{j}]$ m

(2)

(d) Find the distance between the two girls when they are closest together.

(4)

a) Position vector formula : $\boxed{\underline{r} = \underline{r}_0 + vt}$

$\therefore\ \underline{a} = (11i+11j) + t(3i-j)$

b) When $t=6$, $\underline{a} = (11i+11j) + 6\times(3i-j) = (29i+5j)$

$\therefore$ Point P has position vector $(29i+5j)$

To find the value of t when Brionie passes through P we need to equate the position vector of Brionie at time t to the position vector of P.

$\underline{b} = (7i+16j) + t(4i-2j) = 29i+5j$

Compare i and j components :

$4t + 7 = 29 \qquad 4t = 22 \qquad\qquad 16-2t = 5 \qquad 2t = 11$

$t = 5.5$ secs // $\qquad\qquad t = 5.5$ secs //

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Question 6 continued

c) $\vec{AB} = \underline{b} - \underline{a}$

$\vec{AB} = (7i+16j) + t(4i-2j) - [(11i+11j) + t(3i-j)]$

$\vec{AB} = (7+4t)i + (16-2t)j - (11-3t)i - (11-t)j$

$\vec{AB} = (t-4)i + (5-t)j$

d) We should find the magnitude of $\vec{AB}$ to find the closest distance.

$AB^2 = (t-4)^2 + (5-t)^2 = t^2 - 8t + 16 + 25 - 10t + t^2 = 2t^2 - 18t + 41$

The minimum distance occurs when the derivative of the quadratic is zero.

$\frac{d}{dt}(2t^2 - 18t + 41) = 4t - 18 = 0 \quad \therefore \quad 4t = 18 \quad \therefore \quad t = 4.5$ at closest distance.

$\therefore$ Distance at $t=4.5$:

$\vec{AB} = \sqrt{(4.5-4)^2 + (5-4.5)^2} = \sqrt{0.5}\ m \approx 0.707m$ (3sf)

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Question 6 continued

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Question 6 continued

Q6

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7. A helicopter is hovering at rest above horizontal ground at the point H. A parachutist steps out of the helicopter and immediately falls vertically and freely under gravity from rest for 2.5 s. His parachute then opens and causes him to immediately decelerate at a constant rate of 3.9 m s^{-2} for T seconds ($T < 6$), until his speed is reduced to $V\text{ m s}^{-1}$. He then moves with this constant speed $V\text{ m s}^{-1}$ until he hits the ground. While he is decelerating, he falls a distance of 73.75 m. The total time between the instant when he leaves H and the instant when he hits the ground is 20 s.

The parachutist is modelled as a particle.

(a) Find the speed of the parachutist at the instant when his parachute opens. **(1)**

(b) Sketch a speed-time graph for the motion of the parachutist from the instant when he leaves H to the instant when he hits the ground. **(2)**

(c) Find the value of T. **(5)**

(d) Find, to the nearest metre, the height of the point H above the ground. **(4)**

a) Since the parachutist is falling freely under gravity till the parachute is open, SUVAT may be used.

s :

u : 0

v : v

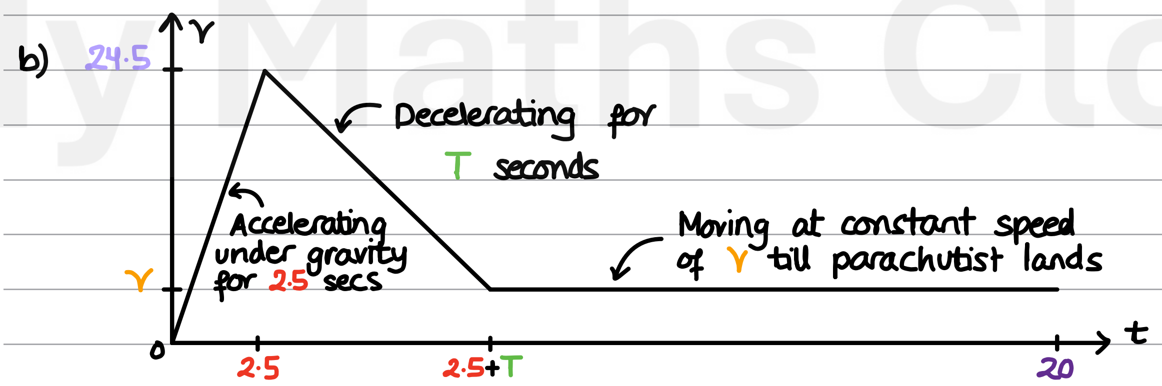
a : 9.8

t : 2.5

Equation without s : $\boxed{v = u + at}$

$v = 0 + 9.8(2.5) = 24.5\text{ ms}^{-1}$ //

b)



c) To calculate T we must set up a SUVAT but only from t = 2.5 to t = 2.5+T as the acceleration must be constant throughout.

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Question 7 continued

s : 73·75
u : 24·5
v : V
a : −3·9
t : T

V is unknown so we can set up another SUVAT to calculate it.

s :
u : 24·5
v : V
a : −3·9
t : T

Equation without s :

$\boxed{v = u + at}$

∴ $V = 24.5 - 3.9T$

s : 73·75
u : 24·5
v : 24·5 − 3·9T
a : −3·9
t : T

As we now have expressions for S, U, V, A and T we can use any SUVAT formula :

$\boxed{v^2 = u^2 + 2as}$ $\boxed{s = ut + \frac{1}{2}at^2}$ $\boxed{s = vt - \frac{1}{2}at^2}$

$\boxed{s = \frac{(u+v)t}{2}}$ $\boxed{v = u + at}$

To lessen the chance of making a small error use the most simple formula that is solvable.

∴ $\boxed{s = \frac{(u+v)t}{2}}$ ∴ $73.75 = \frac{(24.5 + (24.5 - 3.9T))T}{2}$

∴ $147.5 = 24.5T + 24.5T - 3.9T^2$

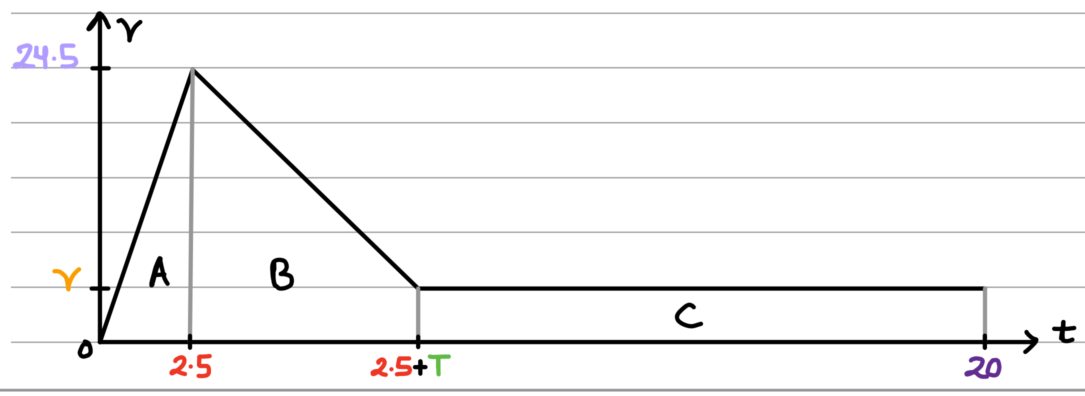
∴ $3.9T^2 - 49T + 147.5 = 0$

∴ $T = 5$ OR $T = 7.564...$

However question states : $T < 6$

∴ $T = 5$ seconds

d) As speed is the rate of change of distance, the area under the speed-time graph equals distance travelled. The area can be seperated into three parts : A, B and C.



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Question 7 continued

$A = \frac{24.5 \times 2.5}{2} = 30.625\,m$ (Using area of triangle formula)

$B = 73.75\,m$ (Already stated in question)

$C = \gamma \times (20 - (2.5 + T)) = 24.5 - 3.9(5) \times (20 - (2.5 + 5)) = 62.5\,m$

$\therefore$ Total height : $H = A + B + C = 30.625 + 73.75 + 62.5 = 166.875\,m$

$\therefore H \approx 167\,m$ (nearest metre)

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Question 7 continued

Q7

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8.

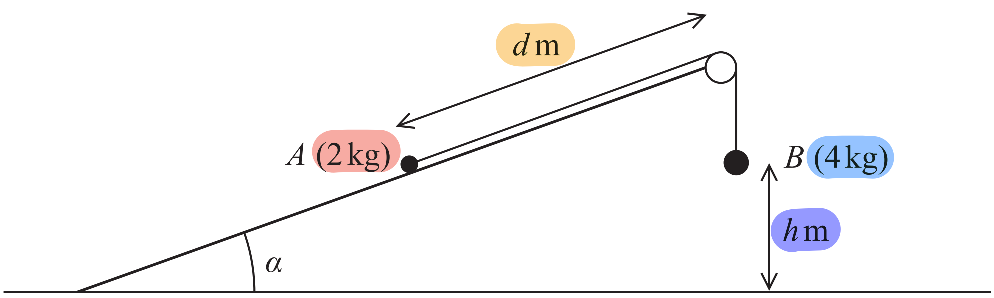


Figure 3

Two particles, A and B, have masses 2 kg and 4 kg respectively. The particles are connected by a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a rough plane. The plane is inclined to the horizontal ground at an angle α where $\tan\alpha = \frac{3}{4}$. The particle A is held at rest on the plane at a distance d metres from the pulley. The particle B hangs freely at rest, vertically below the pulley, at a distance h metres above the ground, as shown in Figure 3. The part of the string between A and the pulley is parallel to a line of greatest slope of the plane. The coefficient of friction between A and the plane is $\frac{1}{4}$

The system is released from rest with the string taut and B descends.

(a) Find the tension in the string as B descends.

(9)

On hitting the ground, B immediately comes to rest.

Given that A comes to rest before reaching the pulley,

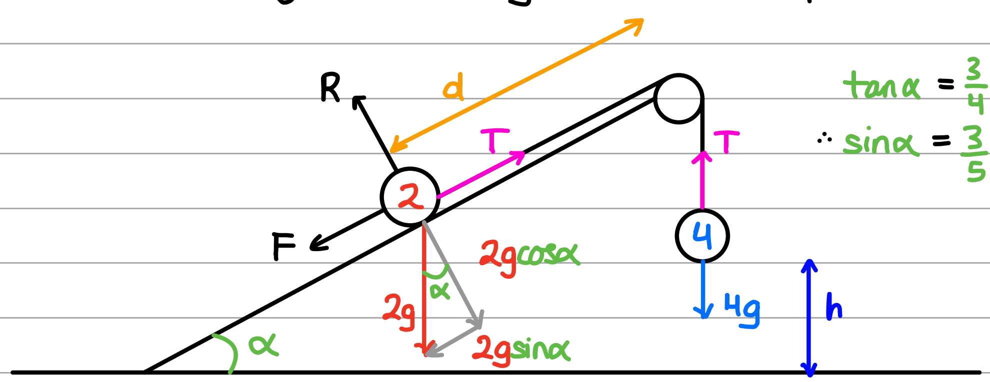
(b) find, in terms of h, the range of possible values of d.

(7)

(c) State one physical factor, other than air resistance, that could be taken into account to make the model described above more realistic.

(1)

a) Draw a diagram labelling all relevant forces.



$\tan\alpha = \frac{3}{4}$

$\therefore \sin\alpha = \frac{3}{5}$, $\cos\alpha = \frac{4}{5}$

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Question 8 continued

Particle A has no motion perpendicular to the plane so the sum of the perpendicular forces is zero.

$\boxed{\Sigma F = 0}$ $\therefore R - 2g\cos\alpha = 0 \therefore R = 2g\cos\alpha = 2(9.8)(0.8) = 15.68\,N$

Equation of motion for the whole system : $\boxed{\Sigma F = ma}$

$\therefore 4g - T + T - 2g\sin\alpha - F = (2+4)a$ Friction formula : $\boxed{F = \mu R}$

$\therefore 4g - 2g\left(\frac{3}{5}\right) - \frac{1}{2}g\left(\frac{4}{5}\right) = 6a$ $\therefore F = \frac{1}{4} \times 2g\cos\alpha = \frac{1}{2}g\cos\alpha$

$\therefore 6a = \frac{12}{5}g$ $\therefore a = \frac{2}{5}g$

Now use the calculated value for acceleration to calculate tension.

Using the equation of motion for particle B only :

$\boxed{\Sigma F = ma}$ $\therefore 4g - T = 4a = 4\left(\frac{2}{5}g\right)$

$\therefore T = 4g - \frac{8}{5}g = \frac{12}{5}g$ //

b) While the string is taut, both particles start from rest with a constant acceleration meaning a SUVAT can be set up.

s: h Equation without t : $\boxed{v^2 = u^2 + 2as}$
u: 0
v: v $\therefore v^2 = 0^2 + 2\left(\frac{2}{5}g\right)h$ $\therefore v^2 = \frac{4}{5}gh$
a: $\frac{2}{5}g$
t: $\therefore v = \sqrt{\frac{4}{5}gh}$ which is the speed of A when B hits the ground.

After particle B hits the ground, the string is slack and A moves up the plane slowing down due to gravity and friction.

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Question 8 continued

Total resistive force on A = $F + 2g\sin\alpha = \frac{2}{5}g + \frac{6}{5}g = \frac{8}{5}g$

$\therefore$ Total deceleration (a_1) : $\boxed{a = \frac{\Sigma F}{m}}$ $\therefore$ $a_1 = \frac{8/5\, g}{2} = \frac{4}{5}g$

Set up a SUVAT from the instant it starts decelerating till rest to calculate the stopping distance of A.

s : s   Equation without t : $\boxed{v^2 = u^2 + 2as}$
u : 0
v : $\sqrt{4/5\, gh}$ $\therefore$ $\frac{4}{5}gh = 0^2 + 2\left(\frac{4}{5}g\right)s$ $\therefore$ $s = \frac{\frac{4}{5}gh}{\frac{8}{5}g} = \frac{h}{2}$
a : $4/5\, g$
t :

$\therefore$ Total distance moved by A = $h + s = h + \frac{h}{2} = \frac{3}{2}h$

The question states A comes to rest before reaching the pulley so d must be greater than the distance travelled.

$\therefore$ Expression for d in terms of h : $d > \frac{3}{2}h$

c) The question doesn't take into account :

- Weight of the string
- Extensibility of the string
- Friction at the pulley

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Question 8 continued

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Question 8 continued

Q8

(Total 17 marks)

TOTAL FOR PAPER: 75 MARKS

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